Petri Nets Tutorial, Parametric Verification
(session 1)

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Thanks

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and of course...

All the developers of the tools
General Introduction
- Why parameters and of what kind?
- Modelling languages: PN, PTA and their extensions.
- Problems of interest.

Parametric Timed Automata
- Basic definitions and examples.
- Decidability results.
- EFSynth and IM algorithms.
- Distributed algorithms.
- IMITATOR in a nutshell.

Parametric Interval Markov Chains
- Basic definitions and examples.
- Algorithm for Parameter Synthesis.
- Detailed example.
General Introduction
First of all...

You know about automata and/or Petri nets:

- about their structure
- about their behaviour
- some analysis techniques
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Nice means to model and analyse concurrent systems...

... but...

- need for tuning the model
- need for parametrisation
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- need for parametrisation

Let us have a deeper look into this now
Why parameters and of what kind?

- **Why parameters?**
  1. **several copies** of a same process or component, dimensioning, e.g.:
     - sensors in a wireless sensor network
  2. **multiple *a priori* possible actions**, e.g.:
     - modelling different design choices
  3. **several hardware characteristics**, e.g.:
     - different response **time** of electronic components
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3. several hardware characteristics, e.g.:
   - different response time of electronic components

What kind of parameters?
1. instances numbering
2. enabled/disabled actions
3. time or probabilities
Usual modelling languages are not sufficient:

- numbering possible with CPN, but fixed *a priori*
- no specific handling of (un)controllable actions
- timing included in TA or TPN, but also fixed
Problems of interest

- model parts of interest with parameters
- find some constraints on parameters guaranteeing desired properties
- find all parameter values guaranteeing these properties
At this stage:

- you have an idea on parametric modelling issues
  - instances
  - (un)controllable actions
  - time or probability constraints
- ...and problems to address
Conclusion

At this stage:

- you have an idea on parametric modelling issues
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Let us start with timing parameters (next sequence)
Parametric Timed Automata: Basic definitions and examples
You have an idea on:

- parametric modelling issues
  - instances
  - (un)controllable actions
  - time or probability constraints
- problems to address
First of all...

You have an idea on:

- parametric modelling issues
  - instances
  - (un)controllable actions
  - time or probability constraints
- problems to address

Let us introduce timing parameters now
Timed automaton (TA)

- Finite state automaton (sets of locations)

Features:
- Location invariant: property to be verified to stay at a location
- Transition guard: property to be verified to enable a transition
- Clock reset: some of the clocks can be set to 0 at each transition
Timed automaton (TA)

- Finite state automaton (sets of locations and actions)

Features:
- Location invariant: property to be verified to stay at a location
- Transition guard: property to be verified to enable a transition
- Clock reset: some of the clocks can be set to 0 at each transition

Example:

\[
\begin{align*}
x &:= 0 \\
y &:= 0 \\
y &:= 5 \\
x &:= 0 \\
y &:= 8 \\
\end{align*}
\]

Transition labels:
- coffee!
- start?
- cup!
- sugar?
Timed automaton (TA)

- Finite state automaton (sets of locations and actions) augmented with a set $X$ of clocks Alur and Dill [1994]
- Real-valued variables evolving linearly at the same rate

```
x := 0
y := 0
```

```
y = 5
```

```
cup!
```

```
x ≥ 1
```

```
sugar?
```

```
tosugar!
```

```
coffee!
```

```
sugar?
```

```
cup!
```

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cup!
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start?
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tosugar!
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  - Real-valued variables evolving linearly at the same rate
  - Can be compared to integer constants in invariants

- Features
  - Location invariant: property to be verified to stay at a location

\begin{align*}
\text{start?} & : x := 0, y := 0, y = 5 \\
\text{sugar?} & : x := 0, y = 8 \\
\text{cup!} & : x \geq 1 \\
\text{coffee!} & : y \leq 5, y \leq 8
\end{align*}
Timed automaton (TA)

- Finite state automaton (sets of locations and actions) augmented with a set $X$ of clocks Alur and Dill [1994]
  - Real-valued variables evolving linearly at the same rate
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- Features
  - Location invariant: property to be verified to stay at a location
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![Diagram of a timed automaton with clocks and transitions](attachment:diagram.png)
**Timed automaton (TA)**

- Finite state automaton (sets of locations and actions) augmented with a set $X$ of clocks Alur and Dill [1994]
  - Real-valued variables evolving linearly at the same rate
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**Features**

- Location **invariant**: property to be verified to stay at a location
- Transition **guard**: property to be verified to enable a transition
- Clock **reset**: some of the clocks can be set to 0 at each transition

```
y ≤ 5
cup!
y = 8
coffee!
y = 5
cup!
```

```
start?
x := 0
y := 0

sugar?
x := 0
```
Concrete semantics of timed automata

- **Concrete state** of a TA: pair \((l, w)\), where
  - \(l\) is a location,
  - \(w\) is a valuation of each clock

- **Concrete run**: alternating sequence of concrete states and actions or time elapse
Examples of concrete runs

Possible concrete runs for the coffee machine

- Coffee with no sugar
  - $x := 0$
  - $y := 0$

- Coffee with 2 doses of sugar
  - $x := 0$
  - $y := 8$

Flowchart:
- Start:
  - $x := 0$
  - $y := 0$
- $y \leq 5$
  - coffee!
  - $x \geq 1$
  - sugar?
  - $x := 0$
- $y = 5$
  - cup!
- $y \leq 8$

Examples of concrete runs

Possible concrete runs for the coffee machine

- **Coffee with no sugar**

  - Start? $x := 0$
  - $y := 0$
  - $y = 5$ cup!
  - $y = 8$ coffee!

- **Coffee with 2 doses of sugar**

  - $x := 0$
  - $y = 8$ coffee!

<table>
<thead>
<tr>
<th></th>
<th>$x$</th>
<th>$y$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>15.4</td>
<td>15.4</td>
</tr>
</tbody>
</table>
Examples of concrete runs

Possible concrete runs for the coffee machine

- Coffee with no sugar

- Coffee with 2 doses of sugar
Decide whether the following properties are satisfied for the timed coffee vending machine

“Once the cup is delivered, coffee will come next within 2 seconds.”

“It is possible to get a coffee with 5 doses of sugar.”

“After the start button is pressed, a coffee is always eventually delivered.”

“It is impossible to press the sugar button twice within 1 second.”
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Why timing parameters?

- Challenge 1: systems incompletely specified
  - Some delays may not be known yet, or may change

- Challenge 2: Robustness Markey [2011]
  - What happens if 8 is implemented with 7.99?
  - Can I really get a coffee with 5 doses of sugar?

- Challenge 3: Optimisation of timing constants
  - Up to which value of the delay between two actions sugar? can I still order a coffee with 3 doses of sugar?

- Challenge 4: Avoid numerous verifications
  - If one of the timing delays of the model changes, should I model check again the whole system?
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- Challenge 4: Avoid numerous verifications
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- A solution: Parametric analysis
  - Consider that timing constants are unknown (parameters)
  - Find good values for the parameters s.t. the system behaves well
Timed automaton (sets of locations, actions and clocks)

Parametric Timed Automaton (PTA)

- **Start?**
  - $x := 0$
  - $y := 0$

- **Sugar?**
  - $x \geq 1$
  - $y = 5$

- **Coffee!**
  - $y = 8$

- **Bounds**
  - $y \leq 5$
  - $y \leq 8$
Parametric Timed Automaton (PTA)

- Timed automaton (sets of locations, actions and clocks) augmented with a set $P$ of parameters Alur et al. [1993]
- **Unknown constants** compared to a clock in guards and invariants

![Diagram of a parametric timed automaton with transitions and initial and final states labeled with conditions involving parameters $p_1$, $p_2$, and $p_3$.]
Conclusion

At this stage:

- you have an idea on Parametric Timed Automata
- and the challenges for parametric analysis
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Let us go for decidability results (next sequence)
Decidability results for Parametric Timed Automata
First of all...

You have an idea on:

- Parametric Timed Automata
- the challenges for parametric analysis
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Let us now see some decidability results
What is decidability?

A decision problem is **decidable** if one can design an algorithm that, for any input of the problem, can answer **yes** or **no** (in a finite time, with a finite memory).
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**Examples:**

“given three integers, is one of them the product of the other two?”

“given a timed automaton, does there exist a run from the initial state to a given location $l$?”

“given a context-free grammar, does it generate all strings?”

“given a Turing machine, will it eventually halt?”
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If a decision problem is undecidable, it is hopeless to look for algorithms yielding exact solutions for computation problems (because that is impossible).
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However, one can:
- design semi-algorithms: if the algorithm halts, then its result is correct
- design algorithms yielding over- or under-approximations
Decision and computation problems for PTA

- **EF-Emptiness** “Does there exist a parameter valuation for which a given location \( l \) is reachable?”
  
  **Example:** “Does there exist at least one parameter valuation for which I can get a coffee with 2 sugars?”

- **EF-Universality** “Do all parameter valuations allow to reach a given location \( l \)?”
  
  **Example:** “Are all parameter valuations such that I may eventually get a coffee?”

- **Preservation of the untimed language** “Given a parameter valuation, does there exist another valuation with the same untimed language?”
  
  **Example:** “Given the valuation \( p_1 = 1, p_2 = 5, p_3 = 8 \), do there exist other valuations with the same possible untimed behaviours?”

- **EF-Synthesis** “Find all parameter valuations for which a given location \( l \) is reachable”
  
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  \[ \sqrt{\text{, e.g. } 0 \leq p_2 \leq p_3 \leq 8} \]

- **EF-Synthesis** “Find all parameter valuations for which a given location \( l \) is reachable”
  
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  \[ 0 \leq p_2 \leq p_3 \leq 8 \]
Decidability for PTA

- **EF-emptiness problem**
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  undecidable

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In fact most interesting problems for PTAs are **undecidable**
Undecidability is achieved for a single parameter

However, reducing the number of clocks yields decidability of the EF-emptiness problem:
Limiting the number of clocks

Undecidability is achieved for a single parameter \cite{Miller2000, Beneš2015}.

However, reducing the number of clocks yields decidability of the EF-emptiness problem:

- 1 parametric clock and arbitrarily many non-parametric clocks and integer-valued parameters \cite{Beneš2015}.
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- 1 parametric clock and arbitrarily many rational-valued parameters \cite{Miller2000}
Limiting the number of clocks

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However, reducing the number of clocks yields decidability of the EF-emptiness problem:

- \(1\) parametric clock and arbitrarily many non-parametric clocks and integer-valued parameters \cite{BenešEtAl2015}.
- \(1\) parametric clock and arbitrarily many rational-valued parameters \cite{Miller2000}.
- \(2\) parametric clocks and \(1\) integer-valued parameter \cite{BundalaAndOuaknine2014}.
L/U-PTA

Definition

A lower/upper bound PTA (L/U-PTA) is a PTA in which each parameter $p$ is always compared with clocks as an upper bound or always as a lower bound.

Lower-bound parameters:

Upped-bound parameters:
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Lower-bound parameters: $p_1, p_3$

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Lower-bound parameters: $p_1, p_3$

Upped-bound parameters: $p_2, p_4$
Decidable problems for L/U-PTA

- **EF-emptiness problem**
  “Does there exist a parameter valuation for which a given location $l$ is reachable?”
  decidable

  Hune et al. [2002]

- **EF-universality problem**
  “Do all parameter valuations allow to reach a given location $l$?”
  decidable

  Bozzelli and La Torre [2009]

- **EF-finiteness problem**
  “Is the set of parameter valuations allowing to reach a given location $l$ finite?”
  decidable (for integer valuations)

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Undecidable problems for L/U-PTA

- **AF-emptiness problem**
  “Does there exist a parameter valuation for which a given location $l$ is always eventually reachable?”
  undecidable

  Jovanović et al. [2015]
Undecidable problems for L/U-PTA

- **AF-emptiness problem**
  “Does there exist a parameter valuation for which a given location \( l \) is always eventually reachable?”
  undecidable
  Jovanović et al. [2015]

- **AF-universality problem**
  “Are all valuations such that a given location \( l \) is always eventually reachable?”
  undecidable (but...)
  André and Lime [2016]
Undecidable problems for L/U-PTA

- **AF-emptiness problem**
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  Jovanović et al. [2015]

- **AF-universality problem**
  “Are all valuations such that a given location \( l \) is always eventually reachable?”
  undecidable (but...)
  André and Lime [2016]

- **language preservation emptiness problem**
  “Given a parameter valuation \( v \), can we find another valuation with the same untimed language?”
  undecidable
  André and Markey [2015]
What can we do with L/U-PTA?

In an L/U PTA, can we syntactically . . .

- use an equality \( (=) \) in a guard or invariant?

- use an equality \( x = p \) in a guard or invariant?
What can we do with L/U-PTA?

In an L/U PTA, can we syntactically . . .

- use an equality (\(=\)) in a guard or invariant?
  yes (without parameters!)

- use an equality \(x = p\) in a guard or invariant?
What can we do with L/U-PTA?

In an L/U PTA, can we syntactically...

- use an equality (=) in a guard or invariant?
  yes (without parameters!)

- use an equality \( x = p \) in a guard or invariant?
  no!
What fits into the class of L/U-PTA?

- Any model with parametric delays given in the form of intervals
  - E.g.: \([p_{\text{min}}, p_{\text{max}}]\)

- Many communication protocols

- All hardware circuits modeled using a bi-bounded inertial delay model
Most interesting problems are undecidable for PTA

…but some become decidable when bounding the number of clocks, or adding restrictions on the use of parameters (L/U-PTA)
Most interesting problems are undecidable for PTA

... but some become decidable when bounding the number of clocks, or adding restrictions on the use of parameters (L/U-PTA)

Let us go for some parameter synthesis algorithms (next sequence)
Parameter synthesis algorithms
First of all...

You know that:

- most problems are undecidable for Parametric Timed Automata
- but some are decidable on specific classes
First of all...

You know that:

- most problems are undecidable for Parametric Timed Automata
- but some are decidable on specific classes

Let us now see some parameter synthesis algorithms
Symbolic states for timed automata

- **Objective**: group all concrete states reachable by the same sequence of discrete actions
- **Symbolic state**: a location $l$ and a (infinite) set of states $Z$
- For timed automata, $Z$ can be represented by a convex polyhedron with a special form called zone, with constraints

\[-d_{0i} \leq x_i \leq d_{i0} \text{ and } x_i - x_j \leq d_{ij}\]

- Computation of successive reachable symbolic states can be performed symbolically with polyhedral operations: for edge $e = (l, a, g, R, l')$:

\[
\text{Succ}((l, Z), e) = (l', (Z \cap g)[R] \cap \text{Inv}(l')) \cap \text{Inv}(l'))
\]

- With an additional technicality there is a finite number of reachable zones in a TA.
Symbolic states for timed automata: Example

\[ \begin{align*}
  y &\leq 4 \\
  x &\geq 2 \\
  y &:= 0
\end{align*} \]

Z_0 = \{ (0,0) \} \uparrow \cap \text{Inv}(\cdot)
Symbolic states for timed automata: Example

\[ y \leq 4 \quad \Rightarrow \quad x \geq 2 \quad \Rightarrow \quad y := 0 \]

\[ Z_0 = \{(0,0)\}^\infty \cap \text{Inv}(\bullet) \]
Symbolic states for timed automata: Example

\[ y \leq 4 \quad x \geq 2 \quad y := 0 \]

\[ Z_0 = \{(0, 0)\}^* \cap Inv(\circ) \]

\[ Z_0 \]
Symbolic states for timed automata: Example

\[ y \leq 4 \quad x \geq 2 \quad y := 0 \]

\[ Z_0 = \{(0, 0)\} \cap \text{Inv}(\bullet) \]

\[ Z_0 \cap (x \geq 2) \]
Symbolic states for timed automata: Example

\[ y \leq 4 \quad x \geq 2 \quad y := 0 \]

\[ Z_0 = \{(0, 0)\} \cap \text{Inv}(\bullet) \]

\[ (Z_0 \cap (x \geq 2))[\{y\}] \]
Symbolic states for timed automata: Example

\[ y \leq 4 \quad \text{and} \quad x \geq 2 \]

\[ y := 0 \]

\[
Z_0 = \{(0, 0)\}^\uparrow \cap \text{Inv}(\bullet)
\]

\[
Z_1 = (Z_0 \cap (x \geq 2))[\{y\}]^\uparrow
\]
Symbolic states for parametric TA

- Symbolic state \((l, Z)\): location + convex polyhedron constraining both clocks and parameters;
- Straightforward extension of reset and future that act only on the clock variables;
- Convex polyhedra obtained have a special form called parametric zone Hune et al. [2002].

There exists in general an infinite number of such symbolic states in a PTA.

\[ Z_0 = \begin{cases} 
    x = y \\
    0 \leq y \leq p \\
    p, q \geq 0
\end{cases} \]

\[ Z_1 = \begin{cases} 
    q \leq x - y \leq p \\
    (q \leq p) \\
    x, y, p, q \geq 0
\end{cases} \]
Symbolic states for parametric TA

- Symbolic state \((l, Z)\): location + convex polyhedron constraining both clocks and parameters;
- Straightforward extension of reset and future that act only on the clock variables;
- Convex polyhedra obtained have a special form called parametric zone Hune et al. [2002].

\[
Z_0 = \begin{cases} 
  x = y & x \geq q \\
  0 \leq y \leq p & y := 0
\end{cases}
\]

\[
Z_1 = \begin{cases} 
  q \leq x - y \leq p & (q \leq p) \\
  x, y, p, q \geq 0
\end{cases}
\]

- There exists in general an infinite number of such symbolic states in a PTA.
A semi-algorithm for parametric reachability

\[
\text{EF}_G(S, M) = \begin{cases} 
Z \downarrow_P & \text{if } l \in G \\
\emptyset & \text{if } S \in M \\
\bigcup_{e \in E} \text{Succ}(S', M \cup \{S\}) & \text{otherwise.}
\end{cases}
\]

- \( S = (l, Z); \)
- \( G \) a set of locations to reach;
- \( M \) is a list of visited symbolic states;
- \( \text{Succ}(S, e) \) computes the symbolic successor of \( S \) by edge \( e \);
- \( \text{EF} \) collects the \textbf{parametric reachability condition} of all symbolic states with a goal location;
- correctness and completeness guaranteed if the algorithm terminates, but...
A semi-algorithm for parametric reachability

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\bigcup_{e \in E} \text{Succ}(S, e) & \text{EF}_G(S', M \cup \{S\})
\end{cases}
\]

if \( l \in G \)
if \( S \in M \)
otherwise.

- \( S = (l, Z) \);
- \( G \) a set of locations to reach;
- \( M \) is a list of visited symbolic states;
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- \( \text{EF} \) collects the \textbf{parametric reachability condition} of all symbolic states with a goal location;
- correctness and completeness guaranteed if the algorithm terminates, but...
- termination is not guaranteed (because the underlying problem is undecidable)
Beyond EFSynth

- EFSynth is the most basic synthesis semi-algorithm for PTA;
- **Termination** can be ensured, using the notion of **integer hull** Jovanović et al. [2015]; André et al. [2015b]:

\[
\begin{array}{c}
\text{at the cost of completeness;} \\
\text{for **bounded** parameters;} \\
\text{but preserves all **integer** points.}
\end{array}
\]

- Similar (semi-)algorithms are also available for more complex properties (e.g. inevitability Jovanović et al. [2015]);
- EFSynth is implemented in IMITATOR and Roméo.
EFSynth is the most basic synthesis semi-algorithm for PTA; 

Termination can be ensured, using the notion of integer hull Jovanović et al. [2015]; André et al. [2015b]:

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Beyond EFSynth

- EFSynth is the most basic synthesis semi-algorithm for PTA;
- **Termination** can be ensured, using the notion of integer hull Jovanović et al. [2015]; André et al. [2015b]:

![Graph](image)

- at the cost of completeness;
- for **bounded** parameters;
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- Similar (semi-)algorithms are also available for more complex properties (e.g. inevitability Jovanović et al. [2015]);
- EFSynth is implemented in IMITATOR and Roméo.
The trace preservation problem

Given a PTA A and a parameter valuation $v_0$, synthesize other valuations yielding the same time-abstract behaviour (trace set). André et al. [2009]; André and Markey [2015]
TPsynth: preserving the untimed behaviour

The trace preservation problem

Given a PTA $A$ and a parameter valuation $v_0$, synthesize other valuations yielding the same time-abstract behaviour (trace set). André et al. [2009]; André and Markey [2015]
TPsynth ("inverse method"): Simplified algorithm

Two parts:

1. Forbid all $v_0$-incompatible behaviours
2. Require all $v_0$-compatible behaviours

Algorithm TPsynth($A, v_0$):
Start with $K_0 = \text{true}$

**REPEAT**

1. Compute a set $S$ of reachable symbolic states under $K_0$

2. Refine $K_0$ by removing a $v_0$-incompatible state from $S$
   - Select a $v_0$-incompatible state $(l, C)$ within $S$ (i.e. $v_0 \not| C$)
   - Add $\neg C \downarrow_P$ to $K_0$

UNTIL no more $v_0$-incompatible state in $S$

RETURN the intersection of all states
An example of flip-flop circuit

- An asynchronous circuit

![Circuit Diagram]

- Concurrent behaviour
  - 4 elements: $G_1, G_2, G_3, G_4$
  - 2 input signals ($D$ and $CK$), 1 output signal ($Q$)

Clarisó and Cortadella [2007]

Question

For these timing delays, does the rise of $Q$ always occur before the fall of $CK$?

Timed model checking gives the answer: yes
An example of flip-flop circuit

- An asynchronous circuit

Concurrent behaviour

- 4 elements: $G_1$, $G_2$, $G_3$, $G_4$
- 2 input signals ($D$ and $CK$), 1 output signal ($Q$)

Timing delays

- Traversal delays of the gates: one interval per gate
An example of flip-flop circuit

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Concurrent behaviour
- 4 elements: \( G_1, G_2, G_3, G_4 \)
- 2 input signals (\( D \) and \( CK \)), 1 output signal (\( Q \))

Timing delays
- Traversal delays of the gates: one interval per gate
- Environment timing constants

Clarísó and Cortadella [2007]
An example of flip-flop circuit

- An asynchronous circuit

  ![Flip-flop diagram](image.png)

  - Concurrent behaviour
    - 4 elements: $G_1$, $G_2$, $G_3$, $G_4$
    - 2 input signals ($D$ and $CK$), 1 output signal ($Q$)
  
  - Timing delays
    - Traversal delays of the gates: one interval per gate
    - Environment timing constants

- Question

  - For these timing delays, does the rise of $Q$ always occur before the fall of $CK$?
An example of flip-flop circuit

- An asynchronous circuit

  ![Circuit Diagram]

  - Concurrent behaviour
    - 4 elements: \( G_1, G_2, G_3, G_4 \)
    - 2 input signals (\( D \) and \( CK \)), 1 output signal (\( Q \))
  
  - Timing delays
    - Traversal delays of the gates: one interval per gate
    - Environment timing constants

- Question
  - For these timing delays, does the rise of \( Q \) always occur before the fall of \( CK \)?
  
  Timed model checking gives the answer: yes
Flip-flop circuit: Timing parameters

Traversal delays of the gates: one interval per gate

4 environment parameters: \(T_{LO}\), \(T_{HI}\), \(T_{Setup}\) and \(T_{Hold}\)

Question: which values of the parameters yield the same untimed behavior as the reference valuation (and hence for which the rise of \(Q\) always occurs before the fall of \(CK\))?
Flip-flop circuit: Timing parameters

- **Timing parameters**
  - Traversal delays of the gates: one interval per gate
  - 4 environment parameters: $T_{LO}$, $T_{HI}$, $T_{Setup}$ and $T_{Hold}$
Timing parameters

- Traversal delays of the gates: one interval per gate
- 4 environment parameters: $T_{LO}$, $T_{HI}$, $T_{Setup}$ and $T_{Hold}$

Question: which values of the parameters yield the same untimed behavior as the reference valuation (and hence for which the rise of $Q$ always occur before the fall of $CK$)?
- **Trace set**: set of all traces of a PTA

- Graphical representation under the form of a tree
  - (Does not give any information on the branching behavior though)
**Trace set**

- Trace set: set of all traces of a PTA
- Graphical representation under the form of a tree
  - (Does not give any information on the branching behavior though)

**Example:** trace set of the flip-flop circuit for the original valuation $v_0$
Application of TPsynth to the flip-flop circuit

\[
\begin{align*}
\nu_0 : & \quad \delta_-^1 = 7 \quad \delta_-^2 = 5 \quad \delta_-^3 = 8 \quad \delta_-^4 = 3 \\
& \quad \delta_+^1 = 7 \quad \delta_+^2 = 6 \quad \delta_+^3 = 10 \quad \delta_+^4 = 7 \\
& \quad T_{HI} = 24 \quad T_{LO} = 15 \quad T_{Setup} = 10 \quad T_{Hold} = 17
\end{align*}
\]

\[K_0 = \text{true}\]
Application of TPsynth to the flip-flop circuit

\[v_0:\]
\[
\delta_1^- = 7 \quad \delta_1^+ = 7 \quad T_{HI} = 24 \\
\delta_2^- = 5 \quad \delta_2^+ = 6 \quad T_{LO} = 15 \\
\delta_3^- = 8 \quad \delta_3^+ = 10 \quad T_{Setup} = 10 \\
\delta_4^- = 3 \quad \delta_4^+ = 7 \quad T_{Hold} = 17
\]

\[K_0 = \text{true}\]

\[D_{\uparrow}\]

\[T_{Setup} \leq T_{LO}\]

\[T_{Setup} \leq T_{LO}\]
Application of TPsynth to the flip-flop circuit

\( \nu_0 : \)
- \( \delta_1^- = 7 \) \( \delta_1^+ = 7 \) \( T_{HI} = 24 \)
- \( \delta_2^- = 5 \) \( \delta_2^+ = 6 \) \( T_{LO} = 15 \)
- \( \delta_3^- = 8 \) \( \delta_3^+ = 10 \) \( T_{Setup} = 10 \)
- \( \delta_4^- = 3 \) \( \delta_4^+ = 7 \) \( T_{Hold} = 17 \)

\( K_0 = \text{true} \)

\( T_{Setup} \leq T_{LO} \)
\( \land \ T_{Setup} \leq \delta_1^+ \)

\( T_{Setup} \leq T_{LO} \)
\( \land \ T_{Setup} \geq \delta_1^- \)
Application of TPsynth to the flip-flop circuit

\[ v_0 : \]
\[
\begin{align*}
\delta_1^- &= 7 & \delta_1^+ &= 7 & T_{HI} &= 24 \\
\delta_2^- &= 5 & \delta_2^+ &= 6 & T_{LO} &= 15 \\
\delta_3^- &= 8 & \delta_3^+ &= 10 & T_{Setup} &= 10 \\
\delta_4^- &= 3 & \delta_4^+ &= 7 & T_{Hold} &= 17 \\
\end{align*}
\]

\[ K_0 = \]
\[ T_{Setup} > \delta_1^+ \]
Application of TPsynth to the flip-flop circuit

\[ \nu_0 : \]

\[ \delta_1^- = 7 \quad \delta_1^+ = 7 \quad T_{HI} = 24 \]
\[ \delta_2^- = 5 \quad \delta_2^+ = 6 \quad T_{LO} = 15 \]
\[ \delta_3^- = 8 \quad \delta_3^+ = 10 \quad T_{Setup} = 10 \]
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\[ K_0 = \]
\[ T_{Setup} > \delta_1^+ \]

\[ T_{Setup} \leq T_{LO} \]
\[ \land T_{Setup} > \delta_1^+ \]
Application of TPsynth to the flip-flop circuit

$v_0 :$

\[
\begin{align*}
\delta_1^- &= 7 & \delta_1^+ &= 7 & T_{HI} &= 24 \\
\delta_2^- &= 5 & \delta_2^+ &= 6 & T_{LO} &= 15 \\
\delta_3^- &= 8 & \delta_3^+ &= 10 & T_{Setup} &= 10 \\
\delta_4^- &= 3 & \delta_4^+ &= 7 & T_{Hold} &= 17
\end{align*}
\]

\[K_0 = T_{Setup} > \delta_1^+\]
Application of TPsynth to the flip-flop circuit

\[ v_0 : \]

\[
\begin{align*}
\delta_1^- & = 7 & \delta_1^+ & = 7 & T_{HI} & = 24 \\
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\end{align*}
\]

\[ K_0 = \]

\[ T_{Setup} > \delta_1^+ \]

\[ T_{Setup} \leq T_{LO} \]
\[ \land T_{Setup} > \delta_1^+ \]

\[ T_{Setup} \leq T_{LO} \]
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\[ T_{Setup} \leq T_{LO} \]
\[ \land T_{Setup} > \delta_1^+ \]

\[ \land T_{Hi} \geq T_{Hold} \]
\[ \land \delta_3^+ \geq T_{Hold} \]
Application of TPsynth to the flip-flop circuit

\[ v_0 : \]
\[
\begin{align*}
\delta_1^- &= 7 & \delta_1^+ &= 7 & T_{HI} &= 24 \\
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\delta_3^- &= 8 & \delta_3^+ &= 10 & T_{Setup} &= 10 \\
\delta_4^- &= 3 & \delta_4^+ &= 7 & T_{Hold} &= 17
\end{align*}
\]

\[ K_0 = \]
\[
T_{Setup} > \delta_1^+ \quad \wedge \quad T_{Hold} > \delta_3^+
\]

\[ T_{Setup} \leq T_{LO} \quad \wedge \quad T_{Setup} > \delta_1^+ \quad \wedge \quad T_{Hold} > \delta_3^+ \]

\[ T_{Setup} \leq T_{LO} \quad \wedge \quad T_{Setup} > \delta_1^+ \quad \wedge \quad T_{Hold} > \delta_3^+ \]

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Application of TPsynth to the flip-flop circuit

\[ v_0 : \]
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\[ k_0 = \]
\[ T_{Setup} > \delta_1^+ \]
\[ \land \quad T_{Hold} > \delta_3^+ \]
Application of TPsynth to the flip-flop circuit

\[ v_0 : \]
\[ \delta_1 = 7 \quad \delta_1^+ = 7 \quad T_{HI} = 24 \]
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\[ \delta_3 = 8 \quad \delta_3^+ = 10 \quad T_{Setup} = 10 \]
\[ \delta_4 = 3 \quad \delta_4^+ = 7 \quad T_{Hold} = 17 \]

\[ K_0 = \]
\[ T_{Setup} > \delta_1^+ \quad \land \quad \delta_3^+ + \delta_4^+ \geq T_{Hold} \]
\[ \land \quad T_{Hold} > \delta_3^+ \quad \land \quad \delta_3^+ + \delta_4^+ < T_{HI} \]
\[ \land \quad T_{Setup} \leq T_{LO} \quad \land \quad \delta_3^+ + \delta_4^+ \leq T_{Hold} \]
\[ \land \quad \delta_1^+ > 0 \]

Diagram showing the flip-flop circuit with transitions and conditions.
Specification and verification of parametric models using parametric timed automata are supported by several software tools:

- **HyTech** (also hybrid automata) [Henzinger et al. 1997]
- **PHAVer** (also hybrid systems) [Frehse 2005]
- **Roméo** (based on parametric time Petri nets) [Lime et al. 2009]
- **IMITATOR** [André et al. 2012]
Two algorithms:

- **EFsynth**: parametric reachability
- **TPsynth**: parametric trace preservation, with a measure of robustness [Markey 2011]

Other algorithms (not presented):

- **AFsynth**: unavoidability synthesis (implemented in Roméo)
- Behavioural cartography (implemented in IMITATOR)

... but all these algorithms are costly.
Conclusion

Two algorithms:
- **EFsynth**: parametric reachability
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Other algorithms (not presented):
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... but all these algorithms are costly.

Let us see how to improve performances with distributed algorithms (next sequence)
Towards Distributed Synthesis Algorithms
First of all...

You have seen some synthesis algorithms for PTA addressing:

- parametric reachability (EFsynth)
- parametric trace preservation (TPsynth)

...but all these algorithms are costly.
First of all...

You have seen some synthesis algorithms for PTA addressing:

- parametric reachability (EFsynth)
- parametric trace preservation (TPsynth)

...but all these algorithms are costly.

Let us now see how to improve performances with distributed algorithms.
Why distributed algorithms?

Algorithms for parameter synthesis for PTA are very **costly**

- time
- memory

Some reasons:

- expensive operations on polyhedra
- no known efficient data structure (such as BDDs or DBMs for timed automata)
Why distributed algorithms?

Algorithms for parameter synthesis for PTA are very costly

- time
- memory

Some reasons:

- expensive operations on polyhedra
- no known efficient data structure (such as BDDs or DBMs for timed automata)

Idea: benefit from the power of clusters

- Cluster: large set of nodes (computers with their own memory and processor)
- Communication between nodes over a network
A first naive approach

Naive approach to distribute EFsynth:
- Each node handles a subpart of the parameter domain
- Each node launches EFsynth on its parameter domain

Drawback: bad performances if the analysis is much more costly in some subdomains than in others
A more elaborate master-worker approach

**Workers**: run a “hybrid” algorithm

- **PRP**: parametric reachability preservation
- Inspired by both EFsynth (to look for bad valuations) and TPsynth (to only explore a limited part of the symbolic state space, while “imitating” a reference valuation)
- Based on integer points: guarantees the coverage of all integer points (but rational-valued points may be missing)

**Master**: responsible for gathering results and distributing reference valuations (“points”) among workers
Master-worker scheme

Master-worker distribution scheme:

- **Workers**: ask the master for a point (integer parameter valuation), calls PRP on that point, and send the result (constraint) to the master
- **Master**: is responsible for **smart repartition** of data between the workers
  - Note: not trivial at all

André et al. [2014, 2015a]
Dynamic domain decomposition

Most efficient distributed algorithm (so far!): “Domain decomposition” scheme

- **Master**
  1. initially splits the parameter domain into subdomains and send them to the workers
  2. when a worker has completed its subdomain, the master splits another subdomain, and sends it to the idle worker

- **Workers**
  1. receive the subdomain from the master
  2. call PRP on the points of this subdomain
  3. send the results (list of constraints) back to the master
  4. ask for more work
Domain decomposition: Initial splitting

- Prevent choosing close points
- Prevent bottleneck phenomenon at the master’s side
  - Master only responsible for gathering constraints and splitting subdomains
Domain decomposition: Dynamic splitting

- Master can balance workload between workers

- Worker2 (Finished)
- Worker1 (Working)
- Master (Splitting)
- Master send split subpart to the worker2
- Worker1 (Calling PRP)
Implementation in IMITATOR

Implemented in IMITATOR using the MPI paradigm (message passing interface)

Distributed version up to 44 times faster using 128 nodes than the monolithic EFsynth

André et al. [2015a]
Conclusion

First version of distributed algorithms for PTA

What remains to be done...?

- Large space for improvement (44 faster with 128 nodes leaves much space for speedup)
- Multi-core parameter synthesis (on a single machine with several processors)
Conclusion

First version of distributed algorithms for PTA

What remains to be done...?

- Large space for improvement (44 faster with 128 nodes leaves much space for speedup)
- Multi-core parameter synthesis (on a single machine with several processors)

Let us see some tool support (next sequence)
IMITATOR in a nutshell
First of all...
First of all...

You now know about:

- Parametric timed automata
- Parameter synthesis algorithms

Let us now see some tool support
IMITATOR

- A tool for modelling and verifying real-time systems with unknown constants modelled with **Parametric Timed Automata**
  - Communication through (strong) broadcast synchronisation
  - Integer-valued discrete variables
  - **Stopwatches**, to model schedulability problems with preemption

- **Verification**
  - Computation of the symbolic state space
  - Parametric model checking (using a subset of **TCTL**)
  - Language and trace preservation, and robustness analysis
  - Parametric deadlock-freeness checking
  - Behavioural cartography
Under continuous development since 2008

A library of benchmarks
- Communication protocols
- Schedulability problems
- Asynchronous circuits
- ... and more

Free and open source software: Available under the GNU-GPL license
IMITATOR

Under continuous development since 2008

A library of benchmarks

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- Schedulability problems
- Asynchronous circuits
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Try it!

www.imitator.fr
Some success stories

- Modelled and verified an asynchronous memory circuit by ST-Microelectronics
  - Project ANR Valmem

- Parametric schedulability analysis of a prospective architecture for the flight control system of the next generation of spacecrafts designed at ASTRIUM Space Transportation
  - Fribourg et al. [2012]

- Solution to a challenge related to a distributed video processing system by Thales

- Formal timing analysis of music scores
  - Fanchon and Jacquemard [2013]
Conclusion

At this stage, you know:

- Parametric timed automata
- synthesis algorithms for timing parameters
Conclusion

At this stage, you know:

- Parametric timed automata
- Synthesis algorithms for timing parameters

But need for parametric probabilities to capture:

- Imprecisions
- Robustness
- Dimensioning

Let us address Markov chains with parameters (next sequence)
Parametric Interval Markov Chains
First of all, you know about:

- parametric timed automata
First of all...

You know about:
- parametric timed automata

Need for parametric probabilities to capture:
- imprecisions
- robustness
- dimensioning
First of all...

You know about:

- parametric timed automata

Need for parametric probabilities to capture:

- imprecisions
- robustness
- dimensioning

Let us now introduce Parametric Interval Markov Chains
Markov Chains (MCs)

An IMC is consistent if it admits at least one implementation.
Interval Markov Chains (IMCs)

An IMC is consistent if it admits at least one implementation.
Interval Markov Chains (IMCs)

Implementation (MC)

Specification (IMC)
Interval Markov Chains (IMCs)

An IMC is consistent if it admits at least one implementation.
Parametric Interval Markov Chains (pIMCs)

Valuating the parameters of $\mathcal{I}$ with valuation $\nu$ gives an IMC $\nu(\mathcal{I})$
Definition

- State $s$ in an IMC is **0-consistent** if there exists a probability distribution over the successors of $s$ that matches the intervals;
- State $s$ in an IMC is **$n$-consistent** ($n \geq 1$) if:
  1. there exists a probability distribution $\rho$ over the successors of $s$ that matches the intervals and
  2. the successors $s'$ such that $\rho(s') > 0$ are $(n - 1)$-consistent.

Theorem

An IMC with $N$ states is consistent if its initial state is $N$-consistent.
Definition

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Theorem

An IMC with $N$ states is consistent iff its initial state is $N$-consistent.
n-consistency for IMCs: first example

Diagram:

- States: 0, 1, 2, 3, 4
- Edges and Annotations:
  - 0 to 1: [0, 1], 0
  - 1 to 2: [0, 0.5]
  - 2 to 3: [0, 0.5]
  - 3 to 4: [0.3, 0.5]
  - 4 to 0: [0.5, 1]
  - 0 to 0: [0.5, 1]
  - 1 to 1: [0.5, 1]
  - 2 to 2: [0.5, 1]
  - 3 to 3: 1
  - 4 to 4: 0.6

- Initial state: 0
- Final state: 4
$n$-consistency for IMCs: first example
$n$-consistency for IMCs: first example

\begin{align*}
\text{0} & \rightarrow \text{1} \\
\text{0} & \rightarrow \text{2} \\
\text{1} & \rightarrow \text{3} \\
\text{2} & \rightarrow \text{4} \\
\text{3} & \rightarrow \text{4} \\
\text{4} & \rightarrow \text{0}
\end{align*}
n-consistency for IMCs: second example
$n$-consistency for IMCs: second example
$n$-consistency for IMCs: second example
At this stage:

- you have an idea on Parametric Interval Markov Chains . . .
- you know how to check consistency for IMCs
Conclusion

At this stage:

- you have an idea on Parametric Interval Markov Chains . . .
- you know how to check consistency for IMCs

Let us see how to check consistency in PIMCs (next sequence)
Checking Consistency in Parametric Interval Markov Chains
You know about:

- the Parametric Interval Markov Chains model
- checking consistency for IMCs
First of all...

You know about:

- the Parametric Interval Markov Chains model
- checking consistency for IMCs

Consistency problem for PIMCs:

- Does there exist a parameter valuation $v$ such that IMC $v(I)$ is consistent?
- Is IMC $v(I)$ consistent for all parameter valuations $v$?
- Compute all parameter valuations $v$ such that IMC $v(I)$ is consistent
You know about:

- the Parametric Interval Markov Chains model
- checking consistency for IMCs

Consistency problem for PIMCs:

- Does there exist a parameter valuation $v$ such that $\text{IMC } v(I)$ is consistent?
- Is $\text{IMC } v(I)$ consistent for all parameter valuations $v$?
- Compute all parameter valuations $v$ such that $\text{IMC } v(I)$ is consistent

Let us now see how to check consistency in PIMCs
Local consistency constraint for state $s$ wrt. some subset $S'$ of its successors:

$$LC(s, S') = \left[ \sum_{s' \in S'} Up(s, s') \geq 1 \right] \cap \left[ \sum_{s' \in S'} Low(s, s') \leq 1 \right] \cap \left[ \bigcap_{s' \in S'} Low(s, s') \leq Up(s, s') \right]$$
\( \text{n-consistency constraints for pIMCs} \)

- **n-consistency constraint for } s \text{ given some cut-off successors:}**
  \[
  \text{Cons}^X_0(s) = LC(s, \text{Succ}(s) \setminus X) \cap [\bigcap_{s' \in X} \text{Low}(s, s') = 0]
  \]
  and for } n \geq 1,
  \[
  \text{Cons}^X_n(s) = \left[\bigcap_{s' \in \text{Succ}(s) \setminus X} \text{Cons}_{n-1}(s')\right] \cap [LC(s, \text{Succ}(s) \setminus X)] \\
  \cap \left[\bigcap_{s' \in X} \text{Low}(s, s') = 0\right]
  \]

- **n-consistency constraint for } s :**
  \[
  \text{Cons}_n(s) = \bigcup_{X \subseteq Z(s)} \text{Cons}^X_n(s)
  \]

\( Z(s) \text{ contains the successors of } s \text{ for which Low is either 0 or a parameter} \)
Theorem (Delahaye et al. [2016])

Given a plMC $I$ with $N$ states and initial state $s_0$, and a parameter valuation $v$:

$v(I)$ is consistent iff $v \in \text{Cons}_N(s_0)$
Consistency for PIMCs: a detailed example

\[(q \leq 0.7) \cap (q \geq 0.3) \cup (q = 1)\]
Consistency for PIMCs: a detailed example

\[(q \leq 0.7) \cap (q \geq 0.3) \cup (q = 1)\]
Consistency for PIMCs: a detailed example

\[
(q \leq 0.7) \cap (q \geq 0.3) \cup (q = 1)
\]
Conclusion

At this stage:

- you know about parametric timed automata, their problems and algorithms
- you know about interval Markov chains with parametric probabilities
Conclusion

At this stage:

- you know about parametric timed automata, their problems and algorithms
- you know about interval Markov chains with parametric probabilities

Let us practice with IMITATOR


References II


