Petri Nets Tutorial, Parametric Verification (session 1)

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Thanks

Thanks for their support to...

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IPI-PAN, IRCCyN, LIPN

and of course...

All the developers of the tools

Outline

- General Introduction
  - Why parameters and of what kind?
  - Modelling languages: PN, PTA and their extensions.
  - Problems of interest.
- Parametric Timed Automata
  - Basic definitions and examples.
  - Decidability results.
  - EFSynth and IM algorithms.
  - Distributed algorithms.
  - IMITATOR in a nutshell.
- Parametric Interval Markov Chains
  - Basic definitions and examples.
  - Algorithm for Parameter Synthesis.
  - Detailed example.


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General Introduction

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You know about automata and/or Petri nets:

- about their structure
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Let us have a deeper look into this now

Why parameters and of what kind?

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  1. several copies of a same process or component, dimensioning, e.g.:
     - sensors in a wireless sensor network
  2. multiple *a priori* possible actions, e.g.:
     - modelling different design choices
  3. several hardware characteristics, e.g.:
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- What kind of parameters?
  1. instances numbering
  2. enabled/disabled actions
  3. time or probabilities

Usual modelling languages are not sufficient:

- numbering possible with CPN, but fixed *a priori*
- no specific handling of (un)controllable actions
- timing included in TA or TPN, but also fixed

Problems of interest

- model parts of interest with parameters
- find some constraints on parameters guaranteeing desired properties
- find all parameter values guaranteeing these properties

Conclusion

At this stage:

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  - instances
  - (un)controllable actions
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- . . . and problems to address

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Let us start with timing parameters (next sequence)


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Parametric Timed Automata: Basic definitions and examples

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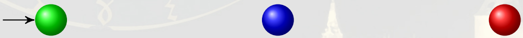
Let us introduce timing parameters now

Timed automaton (TA)

- Finite state automaton (sets of locations)



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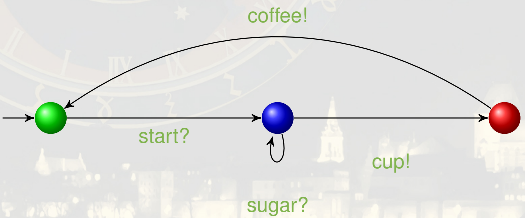

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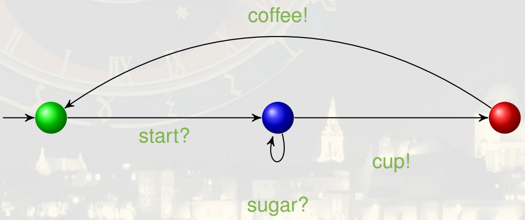

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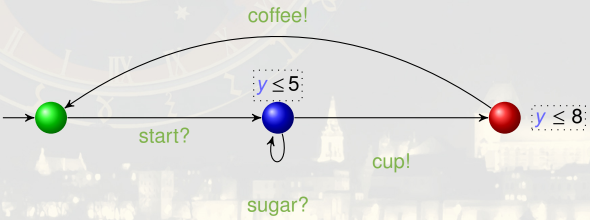
  - Location invariant: property to be verified to stay at a location



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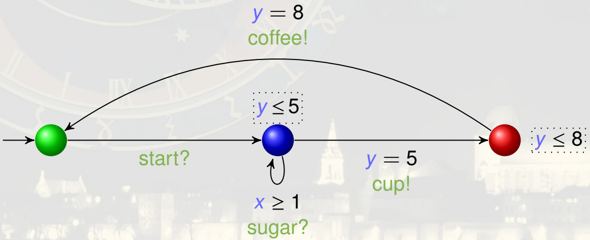

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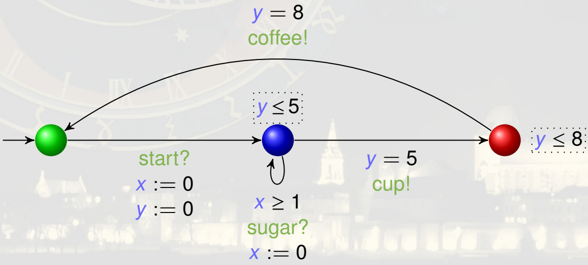
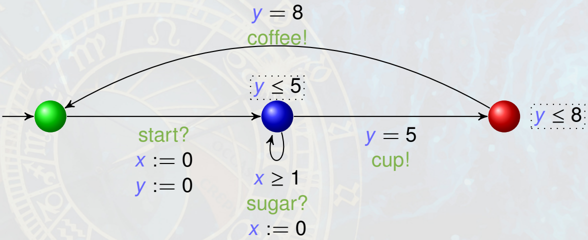
  - Clock reset: some of the clocks can be set to 0 at each transition



Concrete semantics of timed automata

- Concrete state of a TA: pair (l, w), where
  - l is a location,
  - w is a valuation of each clock
- Concrete run: alternating sequence of concrete states and actions or time elapse

Examples of concrete runs



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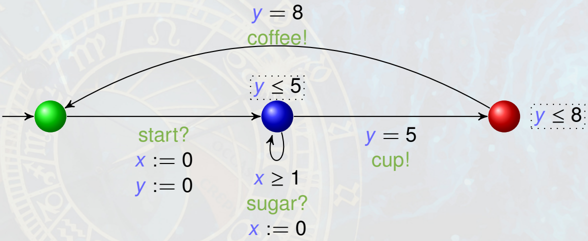

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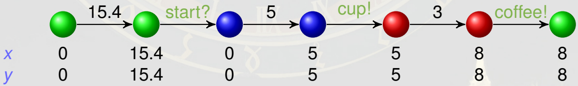

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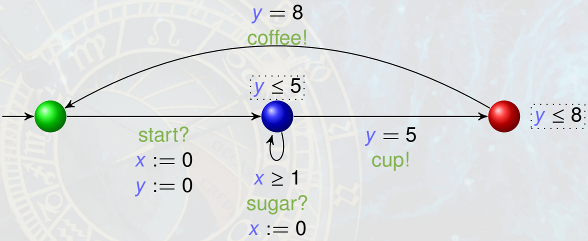

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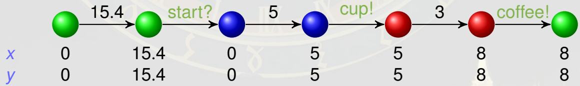
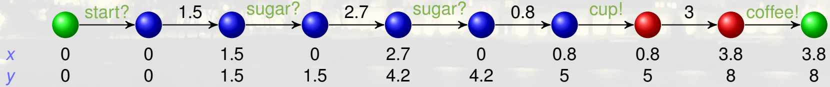


  - Coffee with 2 doses of sugar



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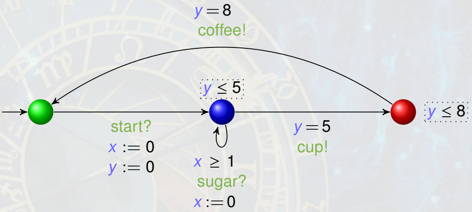

Decide whether the following properties are satisfied for the timed coffee vending machine

> “Once the cup is delivered, coffee will come next within 2 seconds.”
>
> “It is possible to get a coffee with 5 doses of sugar.”
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> “After the start button is pressed, a coffee is always eventually delivered.”
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> “It is impossible to press the sugar button twice within 1 second.”

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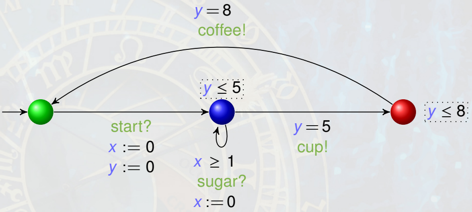

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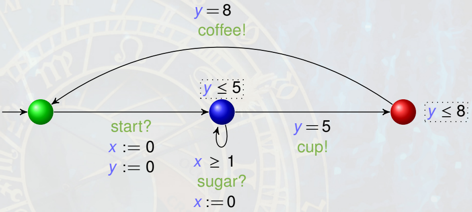

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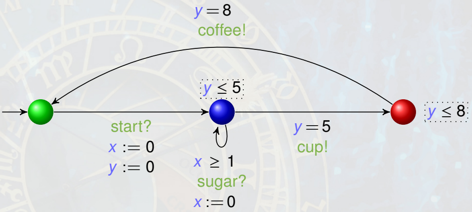

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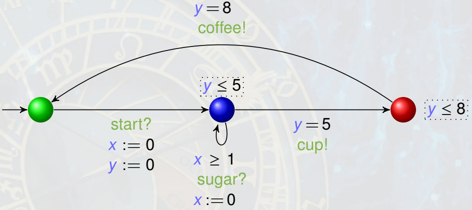

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Why timing parameters?

- Challenge 1: systems incompletely specified
  - Some delays may not be known yet, or may change
- Challenge 2: Robustness Markey [2011]
  - What happens if 8 is implemented with 7.99?
  - Can I really get a coffee with 5 doses of sugar?
- Challenge 3: Optimisation of timing constants
  - Up to which value of the delay between two actions sugar? can I still order a coffee with 3 doses of sugar?
- Challenge 4: Avoid numerous verifications
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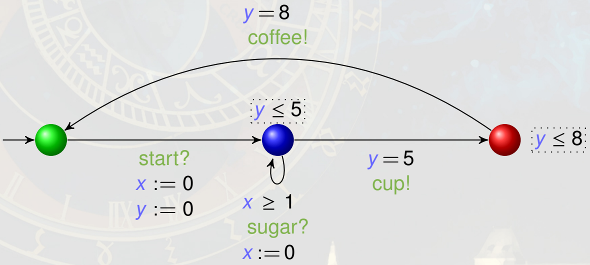
- A solution: Parametric analysis
  - Consider that timing constants are unknown (parameters)
  - Find good values for the parameters s.t. the system behaves well

Parametric Timed Automaton (PTA)

- Timed automaton (sets of locations, actions and clocks)



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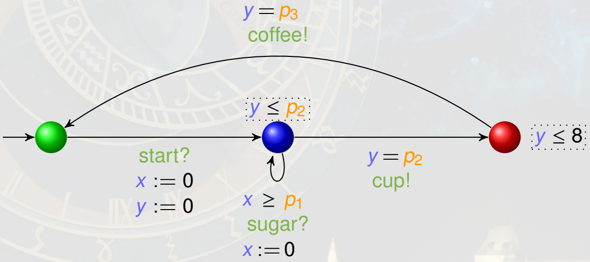

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Let us go for decidability results (next sequence)


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Decidability results for Parametric Timed Automata

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Let us now see some decidability results

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A decision problem is decidable if one can design an algorithm that, for any input of the problem, can answer yes or no (in a finite time, with a finite memory).

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- “given three integers, is one of them the product of the other two?”
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However, one can:

- design semi-algorithms: if the algorithm halts, then its result is correct
- design algorithms yielding over- or under-approximations

Decision and computation problems for PTA

- EF-Emptiness "Does there exist a parameter valuation for which a given location l is reachable?"
  Example: "Does there exist at least one parameter valuation for which I can get a coffee with 2 sugars?"
- EF-Universality "Do all parameter valuations allow to reach a given location l?"
  Example: "Are all parameter valuations such that I may eventually get a coffee?"
- Preservation of the untimed language "Given a parameter valuation, does there exist another valuation with the same untimed language?"
  Example: "Given the valuation $p_1 = 1, p_2 = 5, p_3 = 8$, do there exist other valuations with the same possible untimed behaviours?"
- EF-Synthesis "Find all parameter valuations for which a given location l is reachable"
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In fact most interesting problems for PTAs are undecidable — André [2015]

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- √ 2 parametric clocks and 1 integer-valued parameter Bundala and Ouaknine [2014]

L/U-PTA

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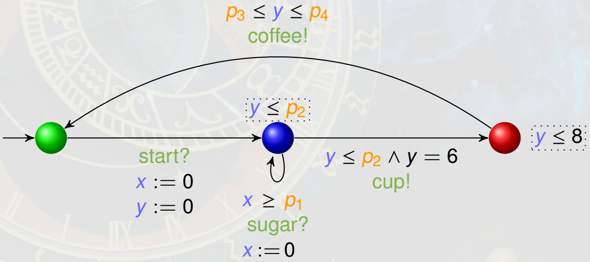

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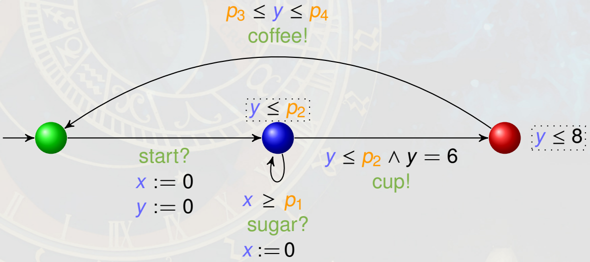

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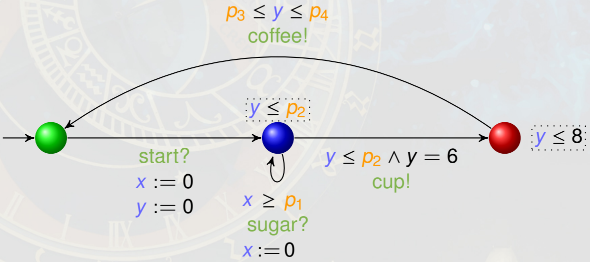

Lower-bound parameters: p_1, p_3

Upped-bound parameters: p_2, p_4

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- EF-finiteness problem
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  decidable (for integer valuations) — Bozzelli and La Torre [2009]

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- language preservation emptiness problem
  "Given a parameter valuation v, can we find another valuation with the same untimed language?"
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What can we do with L/U-PTA?

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  no!

What fits into the class of L/U-PTA?

- Any model with parametric delays given in the form of intervals
  - E.g.: $[p_{min}, p_{max}]$
- Many communication protocols
- All hardware circuits modeled using a bi-bounded inertial delay model

Conclusion

Most interesting problems are undecidable for PTA

…but some become decidable when bounding the number of clocks, or adding restrictions on the use of parameters (L/U-PTA)

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Let us go for some parameter synthesis algorithms (next sequence)


ORTVS

Parameter synthesis algorithms

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Let us now see some parameter synthesis algorithms

Symbolic states for timed automata

- Objective: group all concrete states reachable by the same sequence of discrete actions
- Symbolic state: a location l and a (infinite) set of states Z
- For timed automata, Z can be represented by a convex polyhedron with a special form called zone, with constraints

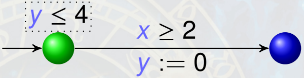
$$-d_{0i} \leq x_i \leq d_{i0} \text{ and } x_i - x_j \leq d_{ij}$$

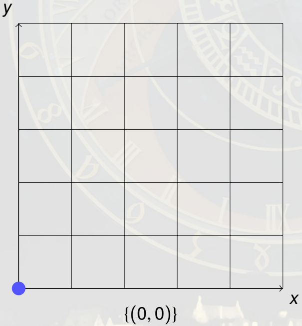
- Computation of successive reachable symbolic states can be performed symbolically with polyhedral operations: for edge $e = (l, a, g, R, l')$:

$$\text{Succ}((l, Z), e) = (l', (Z \cap g)[R] \cap Inv(l'))^{\nearrow} \cap Inv(l'))$$

- With an additional technicality there is a finite number of reachable zones in a TA.

Symbolic states for timed automata: Example





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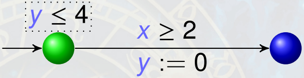

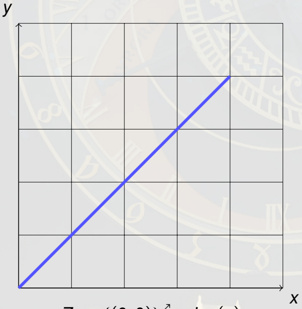

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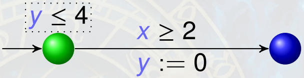

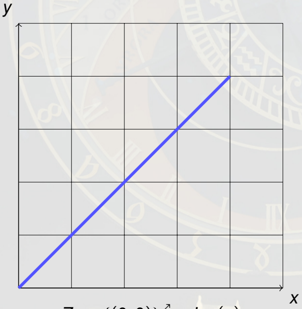

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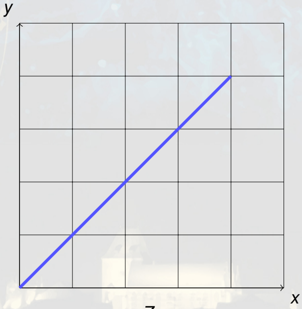

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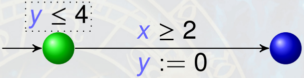

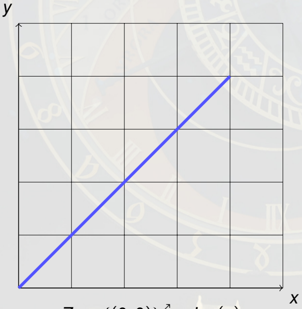

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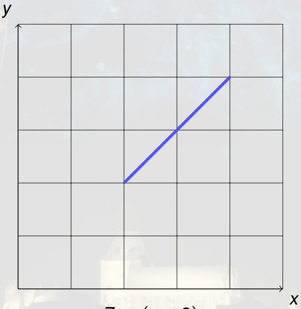

$Z_0 \cap (x \geq 2)$

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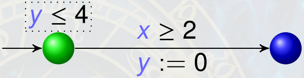

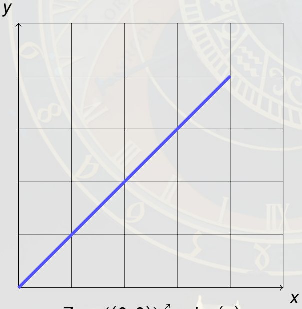

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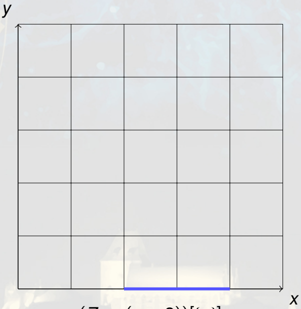

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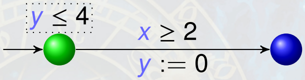

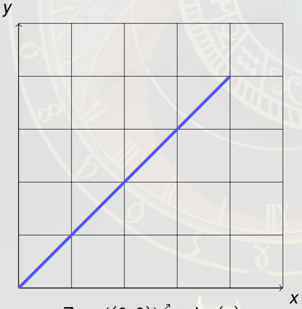

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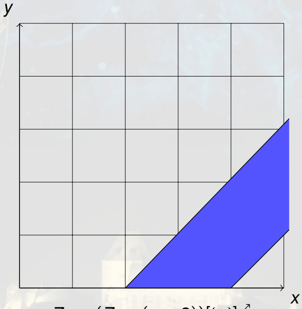
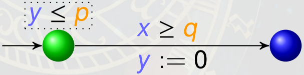


$Z_1 = (Z_0 \cap (x \geq 2))[\{y\}]^{\nearrow}$

Symbolic states for parametric TA

- Symbolic state (l, Z): location + convex polyhedron constraining both clocks and parameters;
- Straightforward extension of reset and future that act only on the clock variables;
- Convex polyhedra obtained have a special form called parametric zone Hune et al. [2002].



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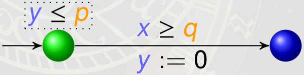

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- There exists in general an infinite number of such symbolic states in a PTA

A semi-algorithm for parametric reachability

$$\mathrm{EF}_G(S, M) = \begin{cases} Z\downarrow_P & \text{if } l \in G \\ \emptyset & \text{if } S \in M \\ \bigcup_{\substack{e \in E \\ S' = \mathrm{Succ}(S,e)}} \mathrm{EF}_G\big(S', M \cup \{S\}\big) & \text{otherwise.} \end{cases}$$

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- G a set of locations to reach;
- M is a list of visited symbolic states;
- $\mathrm{Succ}(S, e)$ computes the symbolic successor of S by edge e;
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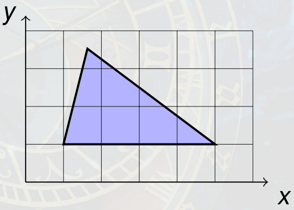
- termination is not guaranteed (because the underlying problem is undecidable)

Beyond EFSynth

- EFSynth is the most basic synthesis semi-algorithm for PTA;
- Termination can be ensured, using the notion of integer hull Jovanović et al. [2015]; André et al. [2015b]:



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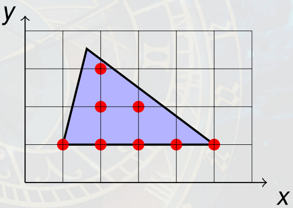

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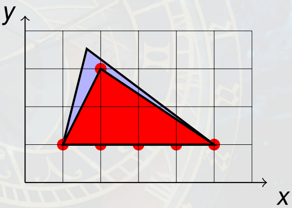

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TPsynth: preserving the untimed behaviour

The trace preservation problem

Given a PTA A and a parameter valuation v_0, synthesize other valuations yielding the same time-abstract behaviour (trace set). André et al. [2009]; André and Markey [2015]

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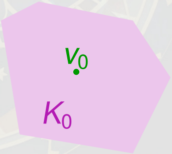


TPsynth ("inverse method"): Simplified algorithm

Two parts:

1. Forbid all v_0-incompatible behaviours
2. Require all v_0-compatible behaviours

Algorithm TPsynth(A, v_0):
Start with $K_0 = \texttt{true}$

REPEAT

1. Compute a set S of reachable symbolic states under K_0
2. Refine K_0 by removing a v_0-incompatible state from S
   - Select a v_0-incompatible state (l, C) within S (i.e. $v_0 \not\models C$)
   - Add $\neg C{\downarrow_P}$ to K_0

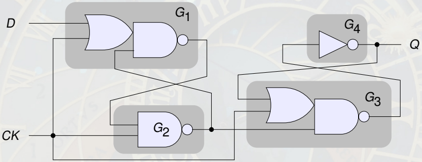
UNTIL no more v_0-incompatible state in S

RETURN the intersection of all states

An example of flip-flop circuit

- An asynchronous circuit

Clarisó and Cortadella [2007]



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  - 4 elements: G_1, G_2, G_3, G_4
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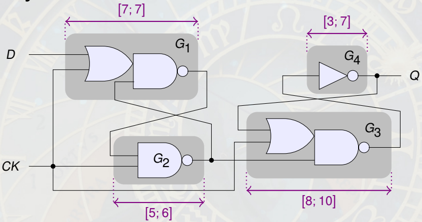

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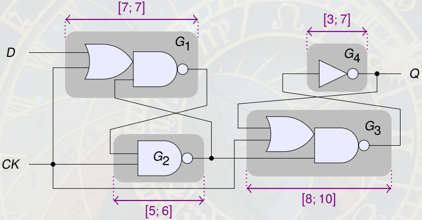

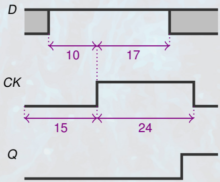

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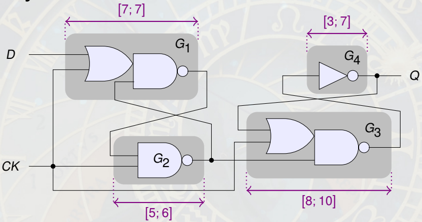

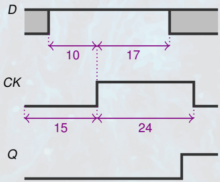

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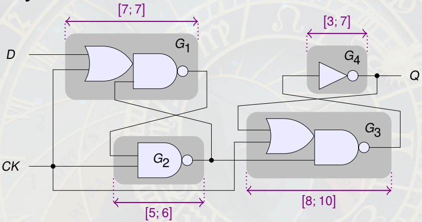

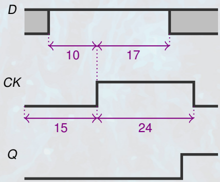

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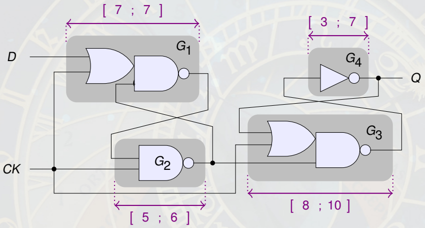
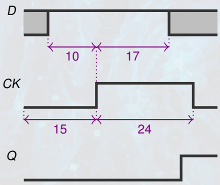
  - Timed model checking gives the answer: yes

Flip-flop circuit: Timing parameters





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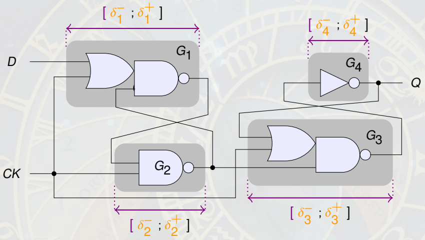

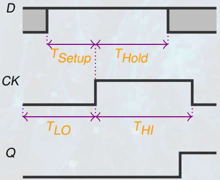

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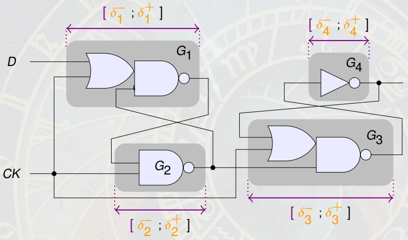

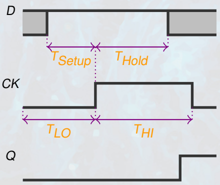

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- Question: which values of the parameters yield the same untimed behavior as the reference valuation (and hence for which the rise of Q always occur before the fall of CK)?

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- Trace set: set of all traces of a PTA
- Graphical representation under the form of a tree
  - (Does not give any information on the branching behavior though)

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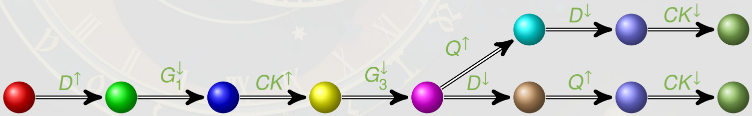
Example: trace set of the flip-flop circuit for the original valuation v_0



Application of TPsynth to the flip-flop circuit

v_0 :

$\delta_1^- = 7$	$\delta_1^+ = 7$	$T_{HI} = 24$
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
$\delta_4^- = 3$	$\delta_4^+ = 7$	$T_{Hold} = 17$

$K_0 = \texttt{true}$

$T_{Setup} \leq T_{LO}$

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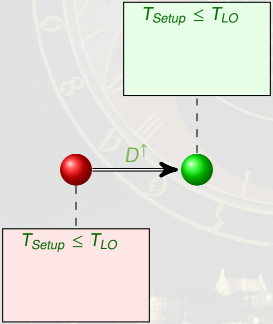

Application of TPsynth to the flip-flop circuit

v_0 :

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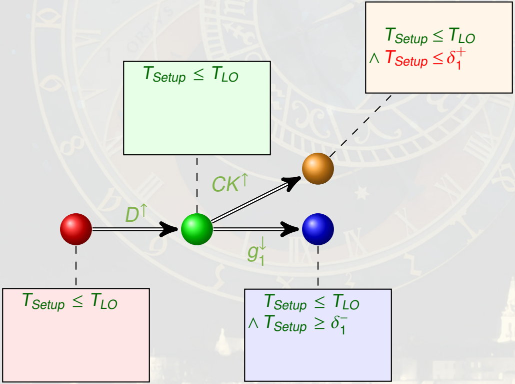

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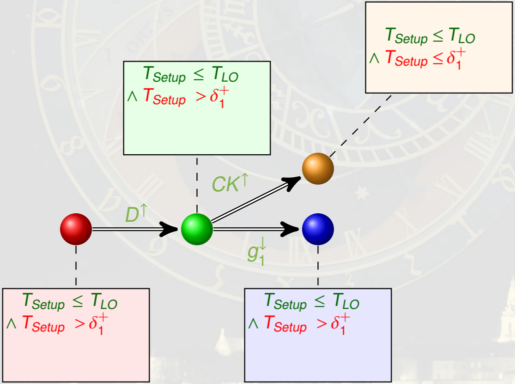

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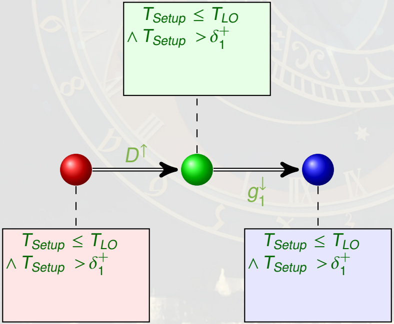

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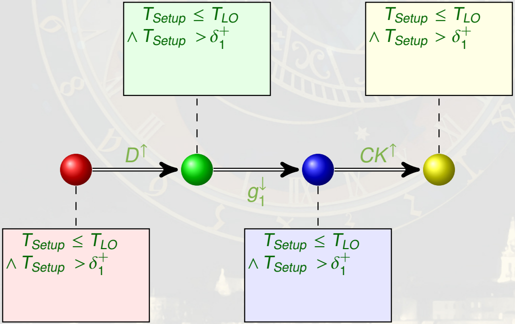

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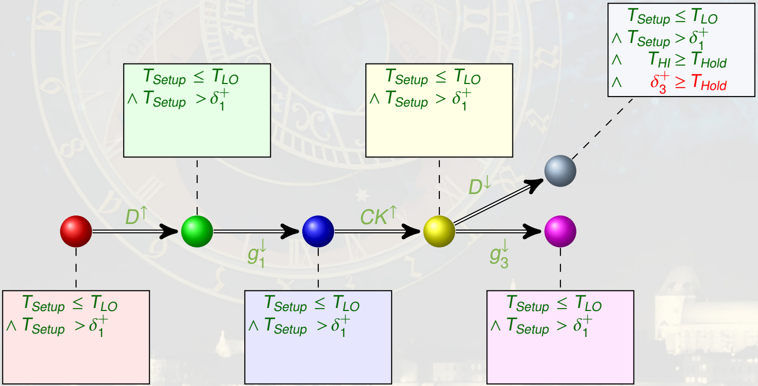

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$$K_0 = T_{Setup} > \delta_1^+ \wedge T_{Hold} > \delta_3^+$$

$$T_{Setup} \leq T_{LO} \wedge T_{Setup} > \delta_1^+ \wedge T_{HI} \geq T_{Hold} \wedge \delta_3^+ \geq T_{Hold}$$

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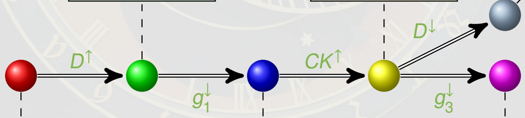

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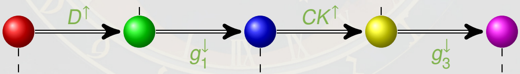

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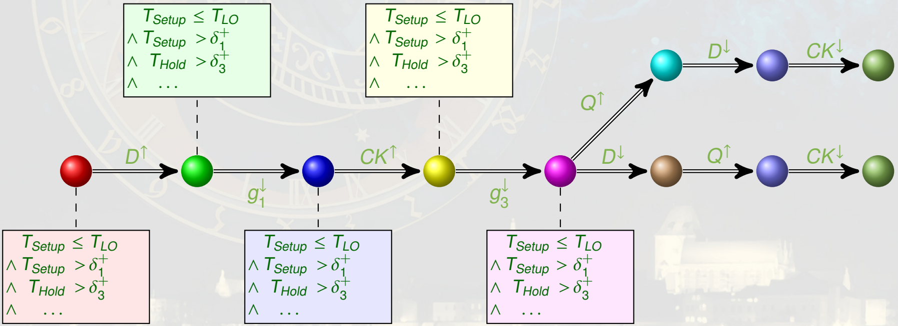
$$K_0 = \begin{array}{ll} T_{Setup} > \delta_1^+ & \wedge\ \delta_3^+ + \delta_4^+ \geq T_{Hold} \\ \wedge\ T_{Hold} > \delta_3^+ & \wedge\ \delta_3^+ + \delta_4^+ < T_{HI} \\ \wedge\ T_{Setup} \leq T_{LO} & \wedge\ \delta_3^- + \delta_4^- \leq T_{Hold} \\ \wedge\ \delta_1^- > 0 & \end{array}$$



Software supporting parametric timed automata

Specification and verification of parametric models using parametric timed automata are supported by several software tools

- HyTech (also hybrid automata) Henzinger et al. [1997]
- PHAVer (also hybrid systems) Frehse [2005]
- Roméo (based on parametric time Petri nets) Lime et al. [2009]
- IMITATOR André et al. [2012]

Conclusion

Two algorithms:

- EFsynth: parametric reachability
- TPsynth: parametric trace preservation, with a measure of robustness Markey [2011]

Other algorithms (not presented):

- AFsynth: unavoidability synthesis (implemented in Roméo)
- Behavioural cartography (implemented in IMITATOR)

...but all these algorithms are costly.

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Let us see how to improve performances with distributed algorithms (next sequence)


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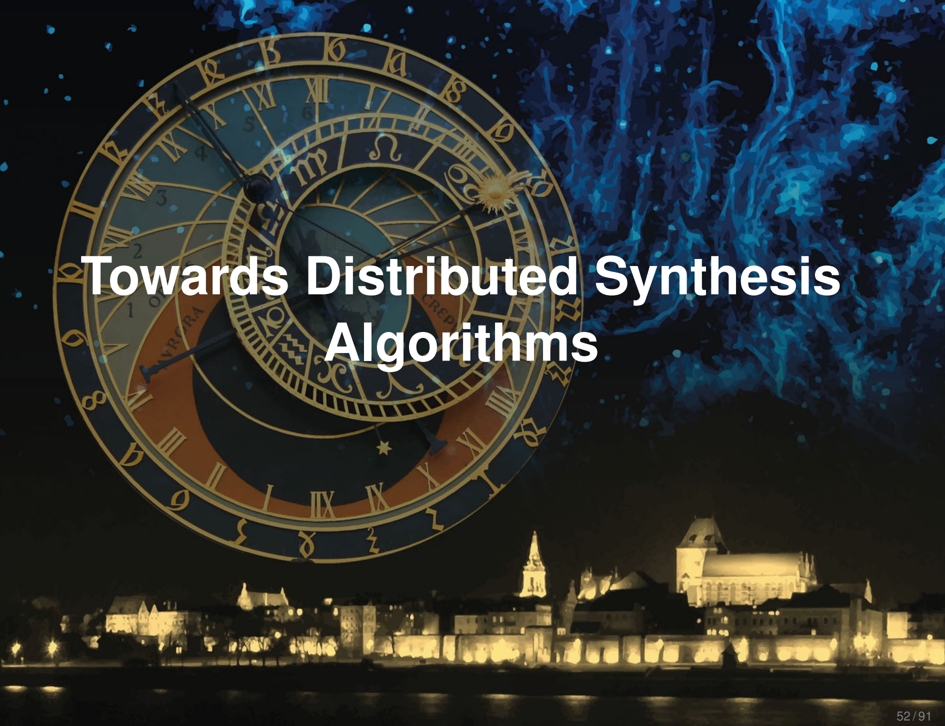
Towards Distributed Synthesis Algorithms



First of all…

You have seen some synthesis algorithms for PTA addressing:

- parametric reachability (EFsynth)
- parametric trace preservation (TPsynth)

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Why distributed algorithms?

Algorithms for parameter synthesis for PTA are very costly

- time
- memory

Some reasons:

- expensive operations on polyhedra
- no known efficient data structure (such as BDDs or DBMs for timed automata)

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- expensive operations on polyhedra
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Idea: benefit from the power of clusters

- Cluster: large set of nodes (computers with their own memory and processor)
- Communication between nodes over a network

A first naive approach

Naive approach to distribute EFsynth:

- Each node handles a subpart of the parameter domain
- Each node launches EFsynth on its parameter domain

Drawback: bad performances if the analysis is much more costly in some subdomains than in others

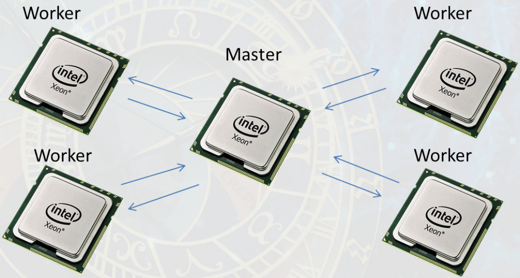
A more elaborate master-worker approach

Workers: run a “hybrid” algorithm

- PRP: parametric reachability preservation
- inspired by both EFsynth (to look for bad valuations) and TPsynth (to only explore a limited part of the symbolic state space, while “imitating” a reference valuation)
- based on integer points: guarantees the coverage of all integer points (but rational-valued points may be missing)

Master: responsible for gathering results and distributing reference valuations (“points”) among workers

Master worker scheme



Master-worker distribution scheme:

- Workers: ask the master for a point (integer parameter valuation), calls PRP on that point, and send the result (constraint) to the master
- Master: is responsible for smart repartition of data between the workers
  - Note: not trivial at all

André et al. [2014, 2015a]

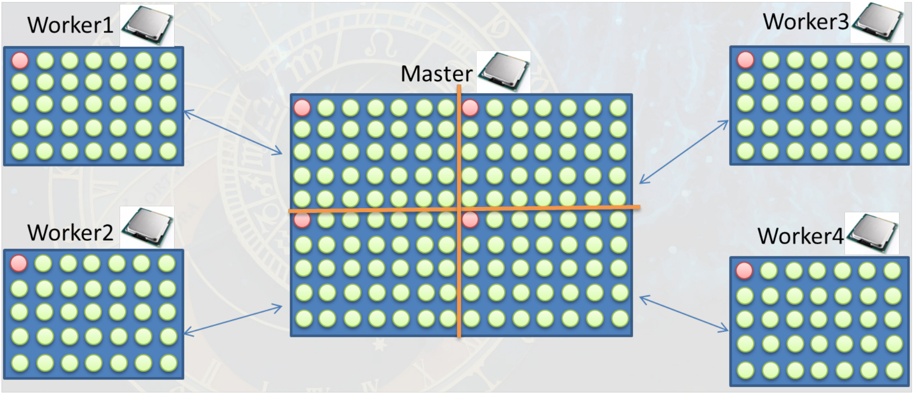
Dynamic domain decomposition

Most efficient distributed algorithm (so far!):
"Domain decomposition" scheme

- Master
  1. initially splits the parameter domain into subdomains and send them to the workers
  2. when a worker has completed its subdomain, the master splits another subdomain, and sends it to the idle worker

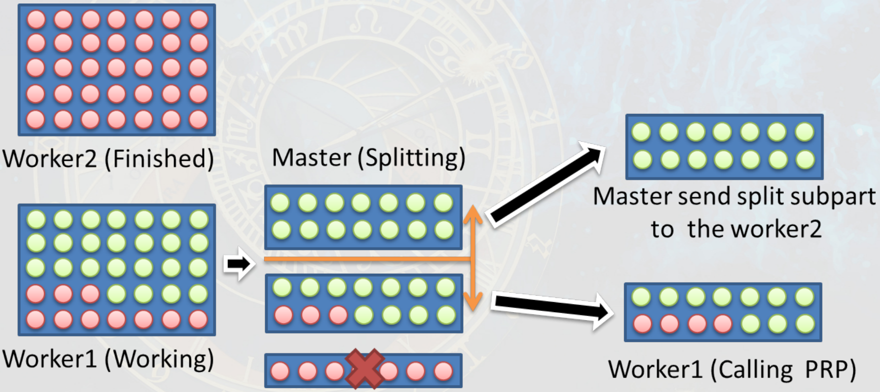
- Workers
  1. receive the subdomain from the master
  2. call PRP on the points of this subdomain
  3. send the results (list of constraints) back to the master
  4. ask for more work

Domain decomposition: Initial splitting



- Prevent choosing close points
- Prevent bottleneck phenomenon at the master's side
  - Master only responsible for gathering constraints and splitting subdomains

Domain decomposition: Dynamic splitting



- Master can balance workload between workers

Implementation in IMITATOR

Implemented in IMITATOR using the MPI paradigm (message passing interface)

Distributed version up to 44 times faster using 128 nodes than the monolithic EFsynth André et al. [2015a]

Conclusion

First version of distributed algorithms for PTA

What remains to be done…?

- Large space for improvement (44 faster with 128 nodes leaves much space for speedup)
- Multi-core parameter synthesis (on a single machine with several processors)

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Let us see some tool support (next sequence)


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IMITATOR in a nutshell

First of all. . .

You now know about:

- Parametric timed automata
- parameter synthesis algorithms

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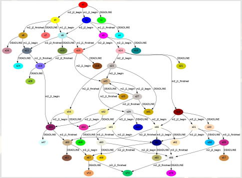
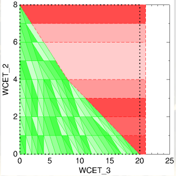
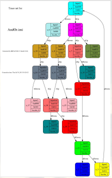
Let us now see some tool support

IMITATOR

- A tool for modelling and verifying real-time systems with unknown constants modelled with Parametric Timed Automata
  - Communication through (strong) broadcast synchronisation
  - Integer-valued discrete variables
  - Stopwatches, to model schedulability problems with preemption
- Verification
  - Computation of the symbolic state space
  - Parametric model checking (using a subset of TCTL)
  - Language and trace preservation, and robustness analysis
  - Parametric deadlock-freeness checking
  - Behavioural cartography







IMITATOR

Under continuous development since 2008 André et al. [2012]

A library of benchmarks

- Communication protocols
- Schedulability problems
- Asynchronous circuits
- . . . and more

Free and open source software: Available under the GNU-GPL license

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Try it!

www.imitator.fr

Some success stories

- Modelled and verified an asynchronous memory circuit by ST-Microelectronics
  - Project ANR Valmem
- Parametric schedulability analysis of a prospective architecture for the flight control system of the next generation of spacecrafts designed at ASTRIUM Space Transportation Fribourg et al. [2012]
- Solution to a challenge related to a distributed video processing system by Thales
- Formal timing analysis of music scores Fanchon and Jacquemard [2013]

Conclusion

At this stage, you know:

- Parametric timed automata
- synthesis algorithms for timing parameters

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but need for parametric probabilities to capture:

- imprecisions
- robustness
- dimensioning

Let us address Markov chains with parameters (next sequence)


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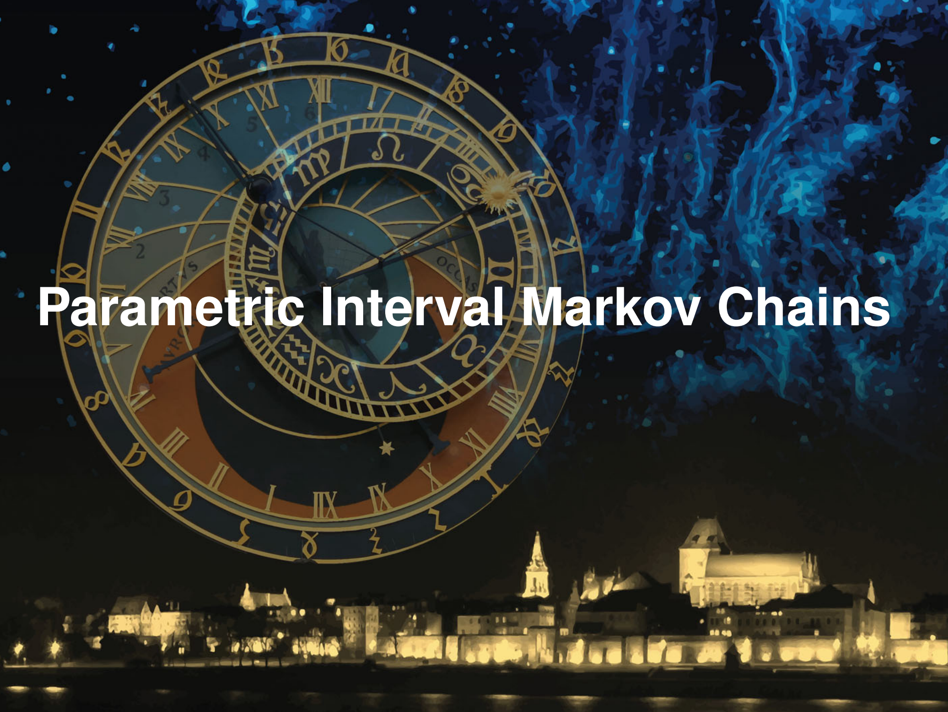
Parametric Interval Markov Chains



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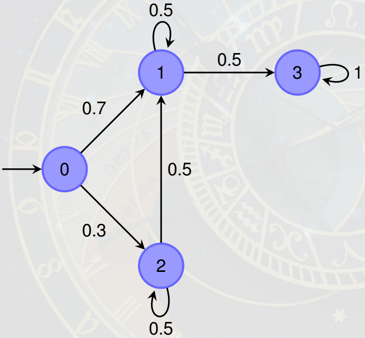
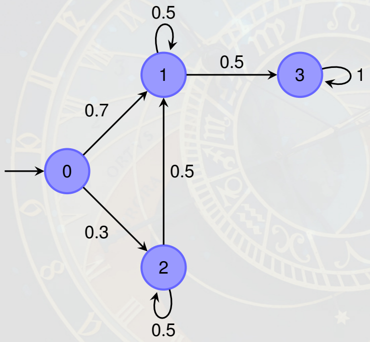
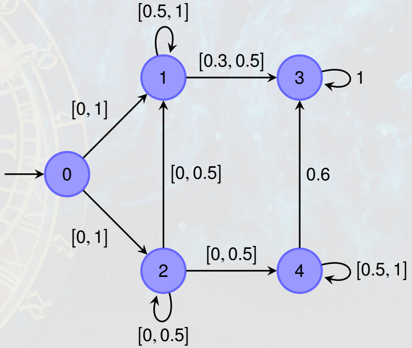
Let us now introduce Parametric Interval Markov Chains

Markov Chains (MCs)



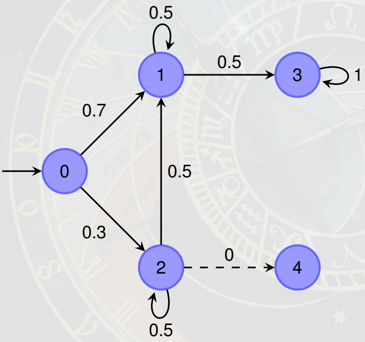
Interval Markov Chains (IMCs)



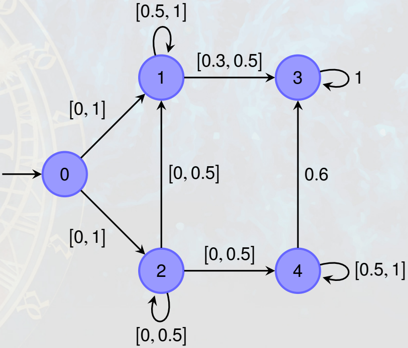


Specification (IMC)

Interval Markov Chains (IMCs)



Implementation (MC)



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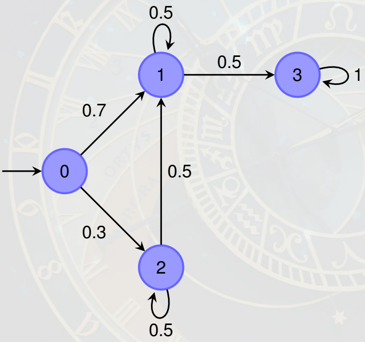

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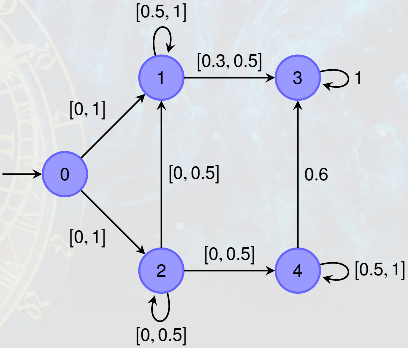
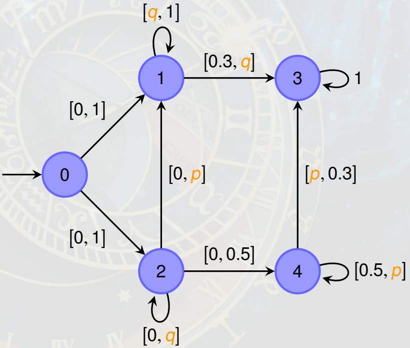


Specification (IMC)

An IMC is consistent if it admits at least one implementation.

Parametric Interval Markov Chains (pIMCs)



Valuating the parameters of $\mathcal{I}$ with valuation v gives an IMC $v(\mathcal{I})$

n-consistency for IMCs

Definition

- State s in an IMC is 0-consistent if there exists a probability distribution over the successors of s that matches the intervals;
- State s in an IMC is n-consistent ($n \geq 1$) if:
  1. there exists a probability distribution ρ over the successors of s that matches the intervals and
  2. the successors s' such that $\rho(s') > 0$ are $(n-1)$-consistent.

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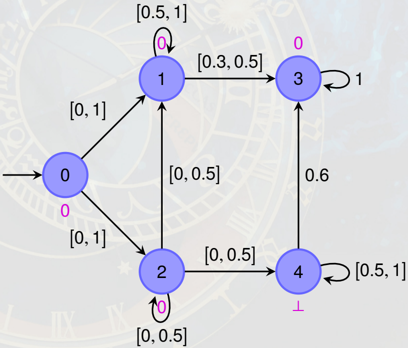
Theorem

An IMC with N states is consistent iff its initial state is N-consistent.

n-consistency for IMCs: first example



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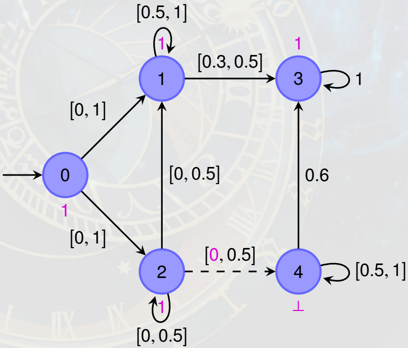

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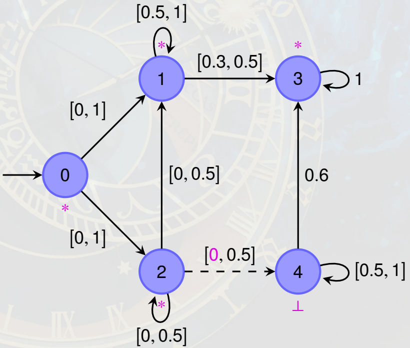

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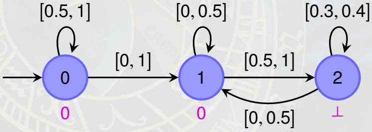

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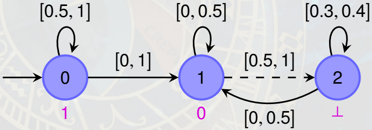

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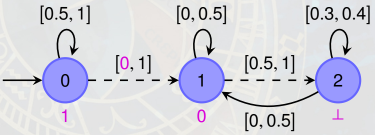

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Let us see how to check consistency in PIMCs (next sequence)


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Checking Consistency in Parametric Interval Markov Chains

First of all…

You know about:

- the Parametric Interval Markov Chains model
- checking consistency for IMCs

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Consistency problem for PIMCs:

- Does there exists a parameter valuation v such that IMC $v(\mathcal{I})$ is consistent?
- Is IMC $v(\mathcal{I})$ consistent for all parameter valuations v?
- Compute all parameter valuations v such that IMC $v(\mathcal{I})$ is consistent

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n-consistency constraints for pIMCs

- **Local consistency constraint for state** s wrt. some subset S' of its successors:

$$LC(s, S') = \left[\sum_{s' \in S'} \mathtt{Up}(s, s') \geq 1\right] \cap \left[\sum_{s' \in S'} \mathtt{Low}(s, s') \leq 1\right] \cap \left[\bigcap_{s' \in S'} \mathtt{Low}(s, s') \leq \mathtt{Up}(s, s')\right]$$

n-consistency constraints for pIMCs

- **n-consistency constraint for s given some cut-off successors**:
  $\mathtt{Cons}_0^X(s) = LC(s, \mathtt{Succ}(s) \setminus X) \cap [\bigcap_{s' \in X} \mathtt{Low}(s,s') = 0]$

  and for $n \geq 1$,

$$\mathtt{Cons}_n^X(s) = \left[\bigcap_{s' \in \mathtt{Succ}(s) \setminus X} \mathtt{Cons}_{n-1}(s') \right] \cap [LC(s, \mathtt{Succ}(s) \setminus X)]$$

$$\cap \left[\bigcap_{s' \in X} \mathtt{Low}(s,s') = 0 \right]$$

- **n-consistency constraint for s**:

$$\mathtt{Cons}_n(s) = \bigcup_{X \subseteq Z(s)} \mathtt{Cons}_n^X(s)$$

$Z(s)$ contains the successors of s for which $\mathtt{Low}$ is either 0 or a parameter

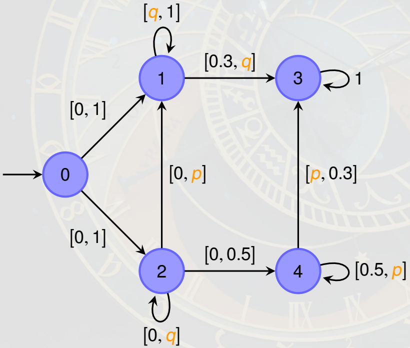
Consistency for pIMCs

Theorem (Delahaye et al. [2016])

Given a pIMC $\mathcal{I}$ with N states and initial state s_0, and a parameter valuation v:

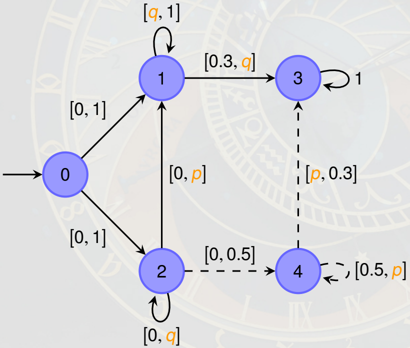
$$v(\mathcal{I}) \text{ is consistent iff } v \in \mathit{Cons}_N(s_0)$$

Consistency for PIMCs: a detailed example



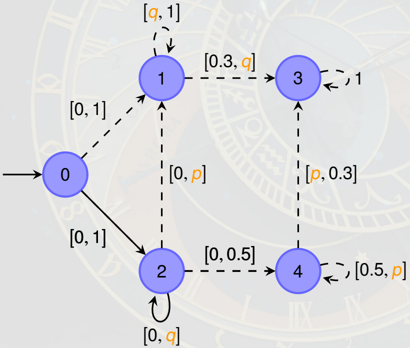
$$[(q \leq 0.7) \cap (q \geq 0.3)] \cup (q = 1)$$

Consistency for PIMCs: a detailed example



$$[(q \leq 0.7) \cap (q \geq 0.3)] \cup (q = 1)$$

Consistency for PIMCs: a detailed example



$$[(q \leq 0.7) \cap (q \geq 0.3)] \cup (q = 1)$$

Conclusion

At this stage:

- you know about parametric timed automata, their problems and algorithms
- you know about interval Markov chains with parametric probabilities

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Let us practice with IMITATOR


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